Received November 26, 1767.

IV. *An Essay on the Connexion between the Parallaxes of the Sun and Moon; their Densities; and their disturbing Forces on the Ocean. By* Patrick Murdoch, *D. D. F. R. S.*

Read Feb. 18, 1768.

IN a letter to Mr. Reid, formerly presented to the Hon. Society, and printed in Vol. LIV. Part II. of the Transactions, mention was made of a rule which I had used for computing the sun's parallax; but as that rule, though it gave a solution near the truth, was in part founded on authority (which, however respectable, ought to be cautiously admitted in such enquiries), I have considered the subject anew, upon such principles alone, as the established theory and the best observations furnished me. And the result is now humbly submitted to the Society.

I. The length of a second-pendulum at the equator, and on a level with the sea, being 36 inches $7\frac{1}{10}$ lines, Paris measure, according to the accurate observations of M. de la Condamine *, that length (properly

* Journal du Voyage, &c. p. 163. In the copy of this paper, which was read in the Society, the length of a second-pendulum, at the equator, had been computed from its length at London: but here it is taken from the immediate observations of a very able astronomer. Some other small alterations have been made; and the examples placed in a better order.

(properly corrected) will, by the reafoning in the abovementioned letter, give the diftance of a moon circulating round an unmoveable earth, equal to 59.95792 femidiameters of the equator. For the logarithm of this number, which is 1.7778438, write l.

Let L be the logarithm of fome greater mean diftance, inferred from obfervations of the moon's parallax; and if r be the natural number to the logarithm $3 \times \overline{L - l}$, and M be taken equal to $\frac{1}{r-1}$, the mafs of the earth will be to that of the moon, as M to 1.

Converfely, If M be any how determined, its equal $\frac{1}{r-1}$, and r, with its logarithm $3 \times \overline{L - l}$ are known; $\frac{1}{3}$ of which is $\overline{L - l}$ to be added to l. For inftance, if, with Sir Ifaac Newton, we put $M = 39.788$, the diftance will be 60.4557, the logarithm L being 1.7814372.

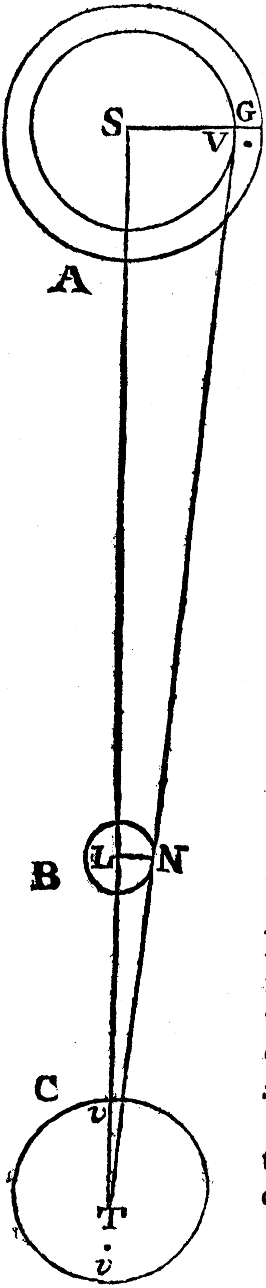
II. If, for each of thefe three, the moon's mafs, her accelerative force on the earth, and her diftance from the earth's centre, we write ($\varphi =$) 1: the accelerative force of the earth on the moon will be reprefented by M, the mafs juft now computed. And if F is the fun's accelerative force on the earth, x his diftance in femidiameters of the lunar orbit, Q the ratio of a fidereal year to a periodic month; we have (by Cor. 2. Prop. 4. Princip. I.) $\frac{F}{x} = \frac{M}{Q^2}$; a given ratio in given terms.

III. The terms F, x, therefore, muſt involve a common factor; by which being divided, the quote may be $\frac{M}{Q^2}$. And this might be obtained innumerable ways, were we to conſider the ratio $\frac{F}{x}$ merely as an abſtract quantity, altogether unreſtricted: it were only putting $M^n \times Q^p = F$. And $\frac{Q^{2+p}}{M^{1-n}} = x$, or $M^{1-n}Q^p = F$, and $\frac{Q^{2+p}}{M^n} = x$: ſo as the ſum of the *indices* of M ſhould be unity, and the difference of thoſe of Q ſhould be 2.

But though the quantities F, x, are as yet unknown, they are not for that indeterminate and variable, as ſuch a liberty of ſubſtitution would import: and all ſubſtitutions which imply the contrary, all *indices* which the theory diſowns, or which are inconſiſtent with obſervation, are to be rejected. In a word, the *indices*, n, p, ought each of them to be *unique*, and determinate (*ſine compare*), * as the quantities F and x are in nature. I take, therefore, $p = 1$, and $n = \frac{1}{2}$; that is, $F = M^{\frac{1}{2}} \times Q$, $x = \frac{Q^3}{M^{\frac{1}{2}}}$. See the examples in the table ſubjoined, upon different ſuppoſitions of the moon's diſtance.

* See Neut. Arith. Univerſal. in the Schol. to Prob. xxiv. The *maxima* and *minima* of variable quantities; the coordinates belonging to a double point, or to a point of reflexion, or contrary flexure, rays of curvature, limits of ratios, &c. All theſe are examples of the *unique*; that is, of quantities in a ſtate that is diſtinguiſhed from and excluſive of all others.

IV. The



IV. The accelerative forces of two ſpherical bodies, A, B, upon a third C, are directly as their maſſes, and inverſely as the ſquares of their central diſtances S T, L T, (or of x and d): which may be thus expreſſed, $\frac{F}{\varphi} = \frac{A}{B} \times \frac{d^2}{x^2}$.

But the maſſes, A, B, being as their reſpective denſities (which call s, m), and the cubes of (R, r), the ſemidiameters of the ſpheres conjunctly; if we write for $\frac{A}{B}$ its equal, we have $\frac{F}{\varphi} = \frac{s}{m} \times \frac{R^3}{r^3} \times \frac{d^2}{x^2}$.

Let the maſs C be to B, as M to 1; and (f) its force on B, will be φ M, or $\varphi = \frac{f}{M}$, which gives $\frac{MF}{f} = \frac{s}{m} \times \frac{x}{d}$, and $\frac{F}{f} = \frac{sx}{Mmd}$: ſuppoſing the apparent ſemidiameters of A and B to ſubtend the ſame angle at the centre of C, and thence R to be to r, as x to d.

V. But if S G, the ſemidiameter of A, be to the ſuppoſed ſemidiameter S V, as q to 1, then, the denſity s remaining, the accelerative force of A (proportional to its magnitude) will be increaſed in the triplicate of that ratio, that is, we now have $\frac{F}{f} = \frac{q^3 s}{M m} \times x$, putting $d = 1$.

And the three bodies, A, B, C, repreſenting the ſun, moon, and earth; likewiſe Q being the ratio of the periods of the earth and moon, it is $\frac{F}{f} = \frac{x}{Q^2}$ (by the corollary quoted in Art. II.). Whence $\frac{s}{m} = \frac{M}{Q^3 \times Q^2}$.

VI. This accelerative force of A remaining, imagine the ſemidiameter S G to be reduced to its former magnitude S V; and the denſity of A will, at the ſame time, be increaſed to $s^1 = q^3 \times s$, and $\frac{s^1}{m} = \frac{M}{Q^2}$.

In which caſe, namely, when the apparent ſemidiameters of A and B (the ſun and moon) are equal, their powers to raiſe a tide at v, a vertex of C, will be as the denſities s', m *: that is, as M the ratio of the earth's maſs to the moon's, and Q^2 the duplicate ratio of the year and month.

Or thus: The diſtances of the bodies, A, B, from the third C, being very great, their powers to raiſe a tide at v, or their diſturbing forces on the ocean, will be directly as their accelerative forces at the centre of C, and inverſely as their diſtances from it: that is, writing a, b, for the diſturbing forces reſpectively; and for the ſun's diſtance in ſemidiameters of C, the Letter z, it will be $a : b :: \frac{F}{z} : \frac{\varphi}{d}$.

* See the laſt page of Dr. Sanderſon's Fluxions: or the late ingenious T. Simpſon's Miſcellaneous Tracts, p. 13.

For,

For, by the general law, a is to F, as the difference of the squares of z and $\overline{z-1}$ is to the square of $\overline{z-1}$; that is, as $2z-1$ to z^2-2z+1. And the same way b is to φ, as $2d-1$ to d^2-2d+1; whence, halving the antecedents, and retaining only z^2, d^2, in the consequents, we have the ratio of a to b, as above.

If the disturbing force of B is exerted at v^1, the opposite vertex of C, b will now be to φ, as $2d+1$ to d^2+2d+1; and, in strictness, we ought to take a mean value of b: but this may be neglected as inconsiderable.

Lastly, let the sun's distance be again expressed in semidiameters of the lunar orbit; that is, if for z we write dx, and unity for φ, we have $a:b::\frac{F}{dx}:\frac{1}{d}$, or as M to Q^2, as before.

VII. In Art. V. it was found that m denoting the density of the moon, and s that of the sun, q^3 being triple the ratio of the sun's mean semidiameter to the moon's *, then will $\frac{m}{s}=\frac{Q^2q^3}{M}$. Whence it will easily follow, that the density of the earth is to that of the sun, as $Q^2\times S^3$ to P^3; P being the moon's horizontal parallax, and S the sun's apparent semidiameter.

* In the *Principia*, the semidiameters are 16′. 6″, and 15′. 38¼″; giving 0 0379755 for the logarithm of q^3. Others take a few seconds from each; which does not much alter the value of q^3.

VIII. I

VIII. I have applied these rules (as in the following Table) to the principal hypothesis of the moon's mean distance.

1°. Supposing it of 60.24 (from Dr. M. Stewart.)
2°. ——————— 60.4 (Cor. 7. Prop. 37. Princip. III.)
3°. ——————— 60.455 (Neut. M being 39.788.)
4°. ——————— 60.493 {(As by Mr. Short's calculations from the transit of ♀.)

The Moon's Mean Distance, being in Semidiameters of the Equator.

		60.24	60.4	60.455	60.493
I	Parall. ☽	′ ″ 57.4,17	′ ″ 56.55 $\frac{1}{16}$	′ ″ 56.52	′ ″ 56.49,88
II.	Mass ♂/☽	70.4225	44.823	39.788	36.9908
III.	☉ dist. to ☽	284.723	356.885	378.293	392.854
IV.	Parall. ☉	″ 12 $\frac{1}{37}$	″ 9,58	″ ‴ 9. $\frac{1}{2}$	″ 8.69
V.	Dens. ☽/☉	2,77 : 1	4,35 : 1	4,9 : 1	5,273 : 1
VI.	Dens. ♂/☽	1,449 : 1	0.9295 : 1	0.827 : 1	0.7707 : 1
VII	Dens. ♂/☉	4.0128 : 1	4.045 : 1	4.0559 : 1	4.06375 : 1
VIII.	Tides ☽/☉	2.5379 : 1	3.9873 : 1	4.5544 : 1	4.8316 : 1
IX.	F. ☉ on ♂ F. ♂ on ☽	1.593 : 1	1.9968 : 1	2.1194 : 1	2.1981 : 1

RE-

REMARKS.

1. If it ſhould be thought that the reaſoning in Art. III. reſts too much upon a metaphyſical principle of Leibnitz, and requires, if not an apology, at leaſt a more formal proof: the ground of ſuch reaſoning, and its extenſive uſe, may be more particularly explained on ſome other occaſion. Suffice it at preſent to add to the note on that article,

That as the given factors (Q and Q^3) in F and x, may be joined with M^n, or with M^{1-n} *indifferently*, the caſe is ſimilar to that of an equal chance at play, for the ſtakes n and $1-n$, where the juſt expectation of each gameſter is $\frac{1}{2}$, whatever be the value of n. The fifth and ſixth Propoſitions of Element I. ſcarce needed any other demonſtration than, that it is manifeſtly impoſſible to aſſign any *reaſon* of inequality, of the angles in one of theſe theorems, and of the ſides in the other. In the following Propoſition, where it is proved, that two lines being extended from the extremities of a right line, A B, to a point C, there is no other point on the ſame ſide of A B, to which lines from A and B equal to the former can be drawn: he who holds the contrary is ſuppoſed to fix upon that other point D; but why D? rather than d, d', d'', &c. he is ſilent: and, therefore, I conclude, there is no ſuch point different from C. And the like may be ſaid of ſome other ſimple theorems, that are commonly demonſtrated by ſhewing the abſurdity of aſſerting their contraries.

2. The title of this paper renders it almoſt needleſs to remind the reader, that the moon's parallax is not here propoſed as the propereſt *medium* for determining that of the ſun. Our *data* are ſtill too uncertain for that purpoſe, ſcarce one of them having been determined to an unexceptionable preciſion; and the numbers in the table ſhew how much a ſmall difference in the moon's diſtance muſt affect the ſeveral concluſions. It may be of uſe, however, to know in what manner thoſe concluſions, as well as the quantities from which they derive *, ſtand related to one another. For if, hereafter, the neceſſary *data* ſhould be more exactly known, the calculus may be repeated; and if the tranſit of Venus, which is to happen in 1769, ſhould confirm Mr. Short's calculations from that of 1759, we may thence conclude the true mean diſtance of the moon, better than in any other way.

3. In the mean time, if any perſon ſhould have the curioſity to examine the numbers of the table, he will pleaſe to take notice:

That, as no two meaſurements, nor any two lengths of a ſecond-pendulum hitherto obſerved, make the earth of the ſame ſpheroid figure, I have retained for the ratio of its greateſt and leaſt diameters, that of 231 to 230; anſwering to the hypotheſis of its uniform denſity: and have thence made a degree of the equator equal to 57200 French toiſes †.

* The connexions of F, x, Q, are manifeſt; and the relation of M to Q^2 is eaſily deduced from Prop. 59. Princip. Book I.

† This was computed upon the ſuppoſition, that a degree of the meridian at lat. $49° \frac{1}{3}$ is 57183 toiſes: but if that degree is, by

The

The meaſure of a ſecond-pendulum, at the equator, I had from M. de la Condamine, as was ſaid before: it was corrected for the centrifugal force, and the reſiſtance of the air; and the moon's diſtance, as revolving round the earth at reſt, was corrected for the ſun's diſturbing force: which is done either by diminiſhing that diſtance in the ſubtriplicate ratio of 178725 to 177725; or by diminiſhing the length of the pendulum in the ſimple ratio of theſe numbers. In this laſt correction I was favoured with the advice and aſſiſtance of the Rev. Mr. Price, and of the Aſtronomer Royal, FF. R. S.

a correction of M. Picart's operations, found to be only 57075 toiſes; the diameter of the equator here uſed ought to be diminiſhed by about 18 10000th parts.

ERRATA.

P. 7. l. 10. *for* confusion *read* concussion.
P. 28. l. 8 *for* $Q^3 \times Q^2$ *read* $q^3 \times Q^2$.
P. 29. *in the note*, *for* 00379755 *read* 90379755.
P. 30. l. 2. *for* hypothesis *read* hypotheses.
Ibid. *throughout the table*, *for* ♂ *read* ♁.
P. 32. l. 16. *for* 1759 *read* 1761.
P. 175. l. 15. *for* being any number *read* z being any number.
P. 176. l. 4. *for* tohat *read* to that.
P. 179. l. 18. *for* $\frac{B}{\frac{1}{3}}$ *read* $\frac{B}{2^{\frac{1}{3}}}$
P. 203. l. 24. *for* thing thing *read* thing.